4. ETUDE COMPARATIVE

Il existe divers modèles "théoriques" permettant de suivre l'évolution du rayon d'une bulle de vapeur sphérique en cours de condensation.

Dans une première étape énous avons effectué la comparaison entre l'évolution du rayon équivalent d'une bulle expérimentale non sphérique et celle d'une bulle sphérique théorique placée dans les mêmes conditions et obéissant à la théorie de Florschuetz et Chao [2]. Ces auteurs ont couplé l'équation de mouvement de Rayleigh avec une solution approchée de l'équation de transfert thermique basée sur l'intégrale de Plesset et Zwick, qui nécessite l'hypothèse d'une mince couche thermique limite, et avec un bilan thermique simplifié à l'interface.

$$R\ddot{R}+\tfrac{3}{2}\dot{R}^2 = \frac{P_{v(T_w)}-P_\infty(t)}{\rho}$$

$$T_w(t)-T_\infty = \left(\frac{\alpha}{\pi}\right)^{1/2}\int_0^t \frac{R^2(x)\dot{R}L\rho_{v(T_w)}}{k\left[\int_x^t R^4(y)\,dy\right]^{1/2}}\,dx$$

avec $k(\partial T/\partial r) = \rho_v(T_w)L\dot{R}$.

Cette comparaison a été faite pour plus de 30 bulles et nous avons toujours mis en évidence la même allure d'évolution pour des bulles axisymétriques. Au contraire les bulles déformées décroissent généralement plus vite et plus irrégulièrement.

Nous fournissons sur la Fig. 5 un exemple caractéristique de cette évolution. On constate (Fig. 5) qu'après un temps d'accord avec la courbe théorique, correspondant à la période initiale d'inertie prépondérante, la courbe expérimentale décroît plus vite que ne le prévoit le modèle. Ajoutons que dans des conditions expérimentales très voisines, Florschuetz et Chao obtiennent eux aussi une différence semblable entre la théorie et l'expérience.

CONCLUSION

Nous avons pu mettre en évidence l'importance des caractéristiques initiales des bulles sur leur évolution. Les petites perturbations mobiles de l'interface et les distorsions de la forme des bulles modifient notablement la condensation et peuvent provoquer leur rupture. Nous avons déterminé les domaines de ruptures liées d'une part à des causes thermiques dues aux transferts préférentiels à l'interface et d'autre part à des causes mécaniques lorsque les bulles principalement soumises à l'inertie du liquide éclatent de façon tout à fait analogue aux bulles de cavitation sous l'effet d'une forte surpression intérieure. En outre, la comparaison de l'évolution du rayon de nos bulles expérimentales à celui de bulles théoriques a permis d'établir que des modèles de bulles sphériques en permanence ne peut fournir que des prévisions grossières du comportement réel des bulles irrégulières.

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THE DEPENDENCE OF LOCAL NUSSELT NUMBER ON PRANDTL NUMBER IN THE CASE OF FREE CONVECTION ALONG A VERTICAL SURFACE WITH UNIFORM HEAT FLUX

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NOMENCLATURE

C^*,	coefficient defined in (1);
Gr_x^*,	modified Grashof number defined by (3);
g,	gravitational acceleration;
Nu_x,	local Nusselt number defined by (2);
Pr,	Prandtl number defined by (4);
q_w,	heat flux at the heated surface;
x,	vertical distance measured from the leading edge of the surface;
α_x,	local heat-transfer coefficient;
β,	coefficient of average thermal expansion;
κ,	thermal diffusivity;
λ,	thermal conductivity;
ν,	kinematic viscosity.

THERE are many solutions of the boundary-layer equations on the free convection along a vertical surface with uniform heat flux, which were obtained by Sparrow–Gregg [1], Gebhart [2], Fujii *et al.* [3], Kuiken [4], Churchill–Ozoe [5] and Ozoe [6]. The local Nusselt number of these solutions, which means the local temperature distribution along the surface, can be expressed by a formula as

$$Nu_x = C^*(Gr_x^* Pr)^{1/5}, \qquad (1)$$

where local Nusselt number Nu_x, modified Grashof number Gr_x^* and Prandtl number Pr are defined by

$$Nu_x = \alpha_x x/\lambda, \qquad (2)$$

$$Gr_x^* = (x^4 g\beta q_w)/\lambda\nu^2, \qquad (3)$$

$$Pr = \nu/\kappa \qquad (4)$$

respectively, and other symbols are given in nomenclature. Coefficient C^* is a function of Pr, and the values for various Pr are shown in Table 1. The object of the present paper is to obtain a simple expression of C^* vs Pr.

Though the significant figures in Table 1 are irregular, every value seems to be sufficiently accurate at least to four decimal places. Therefore, each representative value of C^*

Table 1. The values of C^* obtained from the solutions of the boundary layer equations

Pr	C^*	Authors
0	$0{\cdot}745021 Pr^{1/5}$	Ozoe [6]
0·001	0·1879	Ozoe
0·01	0·288874	Ozoe
0·1	0·41773	Ozoe
	0·41761	Sparrow–Gregg [1]
0·7	0·519180	Kuiken [4]
0·72	0·52054	Fujii *et al.* [3]
1	0·53395	Sparrow–Gregg, Fujii *et al.*
	0·53396	Ozoe, Kuiken
2	0·558363	Kuiken
3	0·570122	Kuiken
4	0·577431	Kuiken
5	0·58304	Fujii *et al.*
	0·582558	Kuiken
7	0·589462	Kuiken
10	0·59587	Sparrow–Gregg, Fujii *et al.*
	0·59588	Ozoe
	0·595803	Kuiken
100	0·61977	Sparrow–Gregg, Fujii *et al.*
	0·61963	Ozoe
	0·61961	Gebhart [2]
1000	0·627749	Ozoe
∞	0·6313	Ozoe

for $Pr \to 0$, $Pr = 0{\cdot}001$, 0·01, ..., 1000 and $Pr \to \infty$ is recommended in the second column of Table 2.

Churchill–Ozoe [5] proposed already a formula as

$$C^* = (0{\cdot}563)^{4/5} \bigg/ \left\{1 + \left(\frac{0{\cdot}437}{Pr}\right)^{9/16}\right\}^{16/45} \tag{5}$$

which is transformed by using the notation of the present paper. This formula is not so accurate in the region of low Pr as shown in the third column of Table 2. On the other hand, C^* may be expressed by such an algebraic function of $Pr^{1/2}$ as that proposed by Le Fevre [7] in the case of uniform surface temperature, for example,

$$C^{*5} = \frac{Pr}{4{\cdot}357 + 8{\cdot}700 Pr^{1/2} + 9{\cdot}973 Pr}. \tag{6}$$

In this equation the coefficients are evaluated by using three values of C^* at $Pr \to 0$, $Pr = 1$ and $Pr \to \infty$, and some numerical values of C^* by (6) are shown in the fourth column of Table 2. The accuracy of (6) is the same order and has the same tendency as that of (5) as shown in the Table.

If the figures in (6) are rounded to the nearest integers, we obtain that

$$C^{*5} = \frac{Pr}{4 + 9Pr^{1/2} + 10Pr}. \tag{7}$$

This is simplest and unexpectedly more accurate than (5) and (6) except for the limit value at $Pr \to 0$, as shown in the fifth column of Table 2. The values calculated by (5) and (6) are about 1 per cent smaller than recommended values in the range of $Pr = 0{\cdot}1$ to 0·01. The former values seem to be affected by the fact that the formulae are made as to satisfy the limit value at $Pr \to 0$. As a conclusion, the authors recommend formula (7) as a simple one which predicts accurate values of C^* for usual Prandtl number.

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Table 2. Comparison among approximate formulae of C^*

Pr	Recommended values	Churchill–Ozoe Formula (5)	Formula (6)	Formula (7)
0	$0{\cdot}7450 Pr^{1/5}$	$0{\cdot}7453 Pr^{1/5}$ (+0·04%)	$0{\cdot}750 Pr^{1/5}$ (exact)	$0{\cdot}7579 Pr^{1/5}$ (+1·7%)
0·001	0·1879	0·1851 (−1·5%)	0·1848 (−1·6%)	0·1877 (−0·1%)
0·01	0·2889	0·2850 (−1·3%)	0·2849 (−1·4%)	0·2885 (−0·1%)
0·1	0·4177	0·4134 (−1·0%)	0·4152 (−0·6%)	0·4179 (+0·05%)
1	0·5340	0·5311 (−0·5%)	0·5340 (exact)	0·5341 (+0·02%)
10	0·5959	0·5969 (+0·2%)	0·5972 (+0·2%)	0·5965 (+0·1%)
100	0·6197	0·6213 (+0·3%)	0·6203 (+0·1%)	0·6197 (0%)
1000	0·6277	0·6287 (+0·2%)	0·6278 (+0·02%)	0·6274 (−0·05%)
∞	0·6313	0·6315 (+0·04%)	0·6313 (exact)	0·6310 (−0·05%)

Percentage errors in the parentheses are calculated in comparison with recommended values.